Closing Tue: 15.1, 15.2
Closing Thu: 15.3, 15.4
Midterm 2 is Tuesday, March 1
It covers 13.3/4, 14.1/3/4/7, 15.1-15.4

15.3 Double Integrals over General Regions

For the rectangular region, *R*, given by

$$a \le x \le b \ , \qquad c \le y \le d$$

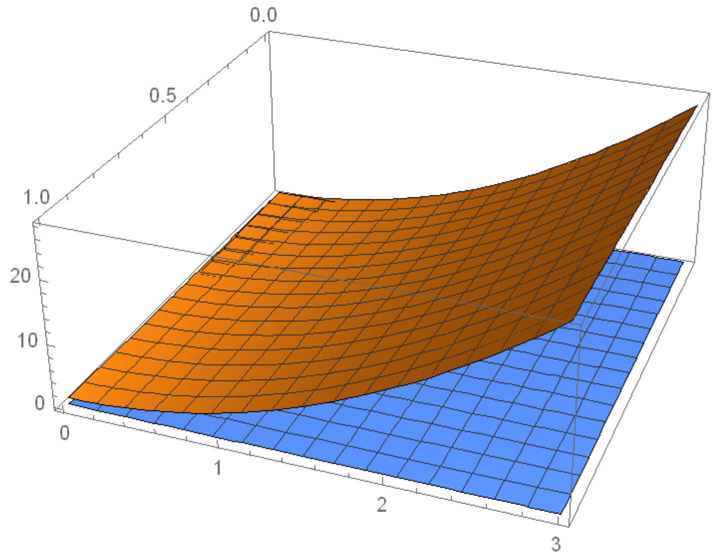
we learned:

$$\iint_R f(x,y)dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$
$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

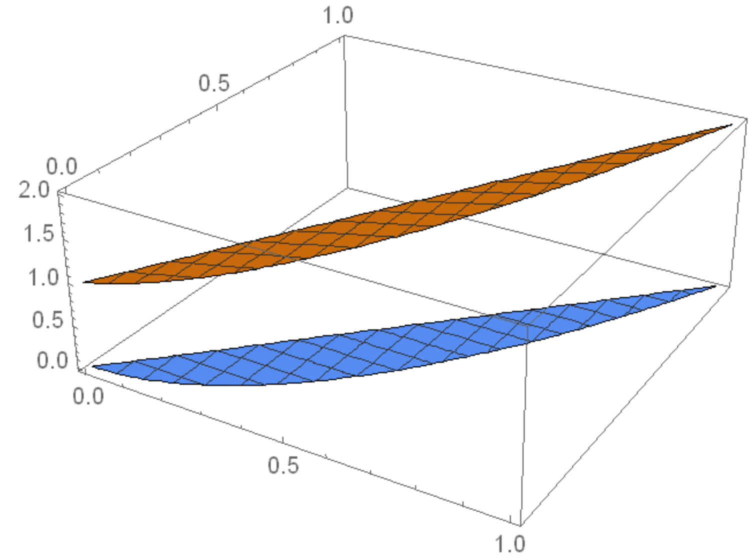
In 15.3, we discuss regions, *R*, other than rectangles.

Type 1 Regions (Top/Bot)	Type 2 Regions (Left/Right)
Given a particular *x* in the range, $a \le x \le b$, we have $g_1(x) \le y \le g_2(x)$	Given a particular *y* in the range, $c \le y \le d$, we have $h_1(y) \le x \le h_2(y)$
$\int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right) dx$	$\int_c^d \left(\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right) dy$

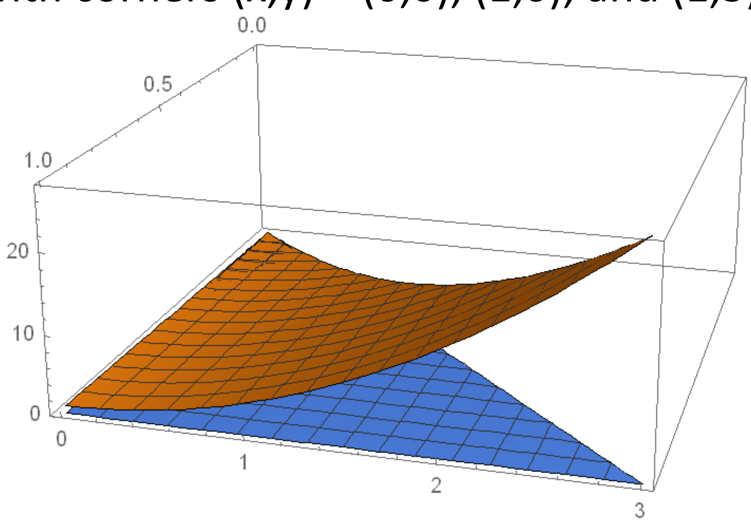
The surface $z = x + 3y^2$ over the rectangular region R = [0,1] x [0,3]



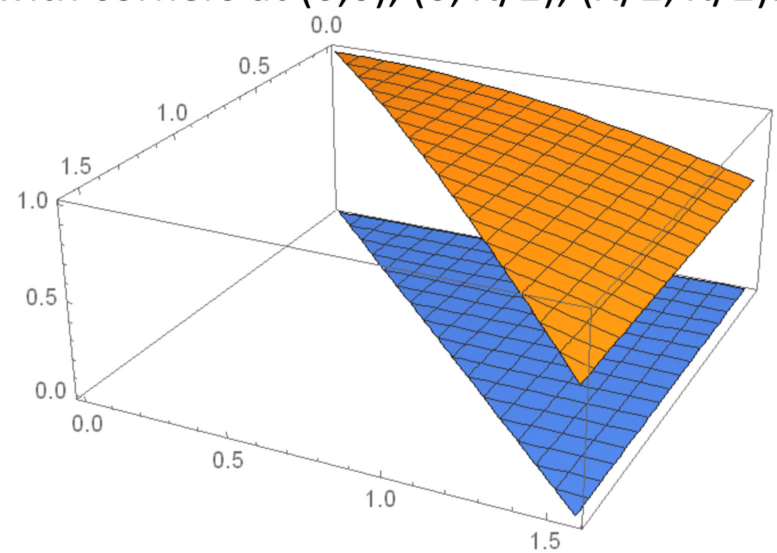
The surface $z = x + 1$ over the region bounded by $y = x$ and $y = x^2$.



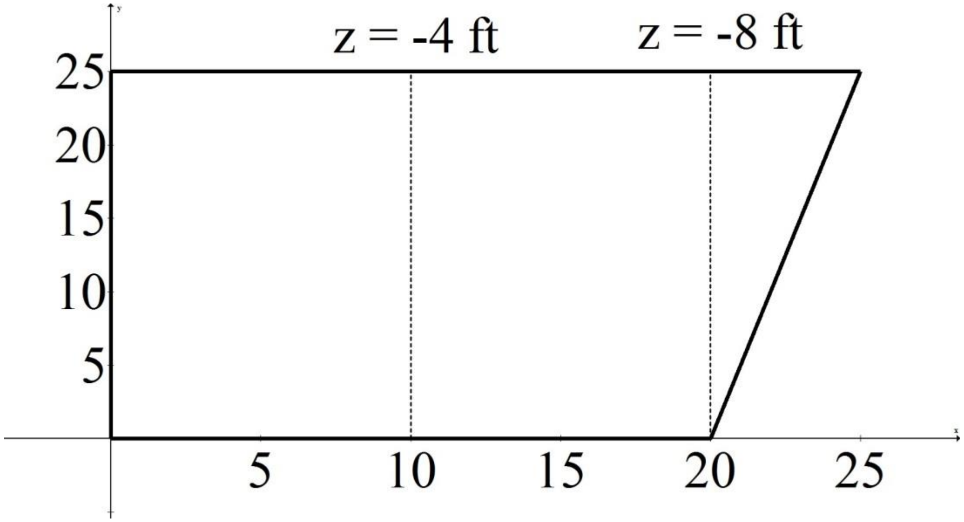
The surface $z = x + 3y^2$ over the triangular region with corners (x,y) = (0,0), (1,0), and (1,3).



The surface $z = \sin(y)/y$ over the triangular region with corners at (0,0), $(0, \pi/2)$, $(\pi/2, \pi/2)$.



An applied problem:
Your swimming pool has the following shape (viewed from above)



The bottom of the pool is a plane with depths as indicated (the pool gets deeper in a linear way from left-to-right)

Solution:

1. Describe surface:
   Slope in y-direction = 0
   Slope in x-direction = -4/10 = -0.4
   Also the plane goes through (0, 0, 0)
   Thus, the plane that describes the bottom of the pool is: **z = -0.4x + 0y**

2. Describe region:
   The line on the right is goes through (20,0) and (25,25), so it has slope = 5 and it is given by the equation
   y = 5(x-20) = 5x – 100
   or **x = (y+100)/5 = 1/5 y + 20**
   The best way to describe this region is by thinking of it as a left-right region.
   On the left, we always have x = 0
   On the right, we always have x = 1/5 y + 20

Therefore, we have

$$\int_0^{25}\left(\int_0^{\frac{1}{5}y+20} -0.4\,x\,dx\right)dy = -741.\overline{6}\ \ \text{ft}^3$$